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Course: Quantitative Methods I

Assignment . Exercise -5.24

Reference: Principle of Econometrics 5th Edition

(A)

|  |  |  |
| --- | --- | --- |
| Dependent Variable: PRICE |  |  |
| Method: Least Squares |  |  |
| Date: 12/14/24 Time: 15:11 |  |  |
| Sample: 1 500 |  |  |  |
| Included observations: 500 |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
| Variable | Coefficient | Std. Error | t-Statistic | Prob.   |
|  |  |  |  |  |
|  |  |  |  |  |
| C | -105.2613 | 13.35759 | -7.880261 | 0.0000 |
| SQFT | 14.91562 | 0.651346 | 22.89969 | 0.0000 |
| SQFT\_AGE | -0.233562 | 0.073423 | -3.181034 | 0.0016 |
|  |  |  |  |  |
|  |  |  |  |  |
| R-squared | 0.648474 |     Mean dependent var | 250.2369 |
| Adjusted R-squared | 0.647059 |     S.D. dependent var | 171.4765 |
| S.E. of regression | 101.8721 |     Akaike info criterion | 12.09130 |
| Sum squared resid | 5157833. |     Schwarz criterion | 12.11658 |
| Log likelihood | -3019.824 |     Hannan-Quinn criter. | 12.10122 |
| F-statistic | 458.4175 |     Durbin-Watson stat | 0.942106 |
| Prob(F-statistic) | 0.000000 |  |  |  |

1**. Constant Term** (C):

- Coefficient: -105.2613

- t-Statistic: -7.880261

- p-value: 0.0000

- Interpretation: The constant term is significantly different from zero at the 1% significance level (p-value < 0.01).

2. **SQFT:**

- Coefficient: 14.91562

- t-Statistic: 22.89969

- p-value: 0.0000

- Interpretation: The coefficient for SQFT is significantly different from zero at the 1% significance level (p-value < 0.01). This indicates a strong positive relationship between SQFT and PRICE.

3. **SQFT\_AGE**:

**Coefficient**: -0.233562

 **t-Statistic**: -3.181034

 **p-value**: 0.0016

- Interpretation: The coefficient for SQFT\_AGE is significantly different from zero at the 1% significance level (p-value < 0.01). This suggests a negative interaction effect between SQFT and AGE on PRICE.

**Overall Model Fit:**

R-squared: 0.648474

 This indicates that approximately 64.85% of the variation in PRICE is explained by the model.

 F-statistic: 458.4175 with a Prob(F-statistic) of 0.000000

The overall model is statistically significant.

In summary, all the estimated coefficients are significantly different from zero, indicating that SQFT and the interaction term SQFT\_AGE have a significant impact on PRICE

(B)

The regression model is given by: PRICE = β1 +β2SQFT+β3(SQFT × AGE) + e

Marginal Effect of SQFT : $\frac{∂E\left(price/X\right)}{∂SQFT}=β2+ β3.AGE\_{}$

The marginal effect of SQFT on the price depends on the value of age. For each additional unit increase in SQFT, the expected change in price is $β2+ β3.AGE\_{}$

Marginal Effect of AGE:: $\frac{∂E\left(price/X\right)}{∂AGE}=β3.AGE\_{}$

The marginal effect of AGE on price depends on the value of SQFT. For each additional unit increase in age, the expected change in price is $β3.sqft$

Assumptions with Categorical AGE

 **Linearity**: The model assumes a linear relationship between PRICE and the interaction term SQFT × AGE.

**Homogeneity**: The effect of SQFT on PRICE is assumed to change linearly with AGE.

 **Ordinal Nature**: AGE is treated as an ordinal variable, assuming a consistent effect across its levels.

( C ) To calculate the marginal effect of SQFT, we use the formula :

 $\frac{∂E\left(price/X\right)}{∂SQFT}=β2+ β3.AGE\_{}$

(Please refer to table above)

Where $β2$= 14.91562

  $β3$= -0.233562

To evaluate for different ages

1. 5 years : $\frac{∂E\left(price\right)}{∂SQFT}$= 14.91562+(-0.233562)\*5= **13.74781**
2. 20 years : $\frac{∂E\left(price\right)}{∂SQFT}$ =14.91562+(-0.233562)\*20= **10.24438**
3. 40 years : : $\frac{∂E\left(price\right)}{∂SQFT}$= 14.91562+(-0.233562)\*40 = **5.57314**

As AGE increases, the marginal effect of SQFT on PRICE decreases, indicating that the value added by additional square footage is less for older houses.

(D )

( i) for 1500 sqft

Ho = -6000

H1≠ 6000

The expected price decline for a house with 1500 square feet is given by the interaction term:

∆price = $β3$\*sqft = -0.233562\*1500

∆price = -350.343

Se( $β3)$ = 0.073423

T o calculate the Test Statistic: : $\frac{∆price-(-6000)}{se( ∆price)}$

Determine the Critical Value for a 5% significance level, the critical value for a two-tailed at ±1.96

t- = $\frac{-350.343-(-6000)}{0.073423\*1500}$

t= 51.50

Since this value is much greater than the critical value of1.96 for a 5% significance level, you would reject the null hypothesis. This suggests that the expected price decline is significantly different from $6000 for a house with 1500 square feet.

( ii) for 3000 sqft

Ho = -6000

H1≠ 6000

Expected price decline

$β3$\*sqft = -0.233562\*3000

Se( $β3)$ = 0.073423=3000.0.073423

t- =$\frac{-700.686-(-6000)}{220.269}$

t$ ≈$ 24.05

The t-statistic is approximately 24.05, which is much greater than 1.96. Reject the null hypothesis. The expected price decline is significantly different from $6000 for a house with 3000 square feet

( iii) for 4500 sqft

Ho = -6000

H1≠ 6000

Expected price decline

$β3$\*sqft = -0.233562\*4500

Se( $β3)$ = 0.073423=4500\*.0.073423

t- =$\frac{-1051.029-(-6000)}{330.4035}$

t$ ≈$ 14.98

The t-statistic is approximately 14.98, which is much greater than 1.96. Reject the null hypothesis. The expected price decline is significantly different from $6000 for a house with 4500 square feet

( E )
Using the regression equation:
PRICE=β1+β2⋅SQFT+β3⋅(SQFT×AGE)

 Substitute the Values:
 AGE = 60
SQFT = 2500
Interaction Term: SQFT\_AGE=2500×60

 β1=−105.2613
 β2=14.91562
 β3=−0.233562

 Predicted Price:
Predicted PRICE=−105.2613+14.91562⋅2500+(−0.233562)⋅(2500×60)

Predicted PRICE= $2149.4887

Standard Error of Prediction:

Se(prediction) = $\sqrt{se^{2}+var(predicted price)}$

Prediction Interval:
PI=Predicted PRICE±tα/2,n−2×SE(prediction)

- tα/2,n−2 is the critical value from the t-distribution for a 95% confidence level.

 the Interval:
SE(prediction) is approximately the standard error of the regression for simplicity. - PI=2149.4887±1.96×101.8721
 PI=2149.4887±199.6733

 Prediction Interval:
- Lower Bound: 2149.4887−199.6733=1949.8154
- Upper Bound: 2149.4887+199.6733=2349.162